

# ANISOTROPY AND SUPERCONDUCTIVITY

Boris V. Bondarev

*Moscow Aviation Institute, Volokolamskoye Shosse 4, 125871, Moscow, Russia*

E-mail: bondarev.b@mail.ru

The mean field method is applied for analysis of valence electrons in metals. It is shown that at low temperatures electrons have two wave-vector distribution patterns. Isotropic distribution refers to the first pattern. Anisotropic distribution refers to another pattern, particularly to specific wave-vector values occurred nearby the Fermi sphere. It is shown that it is the anisotropy that makes the metal obtain its specific superconductor features.

## 1. Introduction

Prior to discussing the matters disclosed in scope of this document, I will give my account of the new model Hamiltonian, the Nobel Prize Winner, P. W. Anderson addressed to in his lecture. This Hamiltonian is now named, as Anderson Hamiltonian. Hereunder, I'd like to give some extracts from the above [1, 2].

"To describe the case, I put forward the model Hamiltonian, which is now referred to as Hubbard Hamiltonian:

$$H = \sum_{i,j} b_{ij} c_{i\sigma}^+ c_{j\sigma} + \sum_i U n_{i\uparrow} n_{i\downarrow},$$

where  $b_{ij}$  describes the  $i$  – to –  $j$  node electron jump amplitude and  $U$  – repulsion energy of two single-centered opposite-spin electrons. Need to say, that no single-centered parallel-spin states are possible.

It is assumed that there is a local atomic spin denoted by  $\mathbf{S}$  and existent from the God. It is also assumed that this kind of a spin interacts with free electrons on their exchangeable basis. This very statement is exactly referred to Kondo Hamiltonian:

$$H = \sum_{k,\sigma} \varepsilon_k n_{k\sigma} + J \mathbf{S} \mathbf{s},$$

where  $J$  is an empirically supported exchange integral,

$$\mathbf{s} = \sum_{k,k',\sigma,\sigma'} c_{k\sigma}^+ \sigma_{\sigma\sigma'} c_{k'\sigma'}$$

– local spin density of impurity conductivity electrons.

Anderson model is a very simple one to describe the aforesaid electronic mechanism. We introduce critically important single-centered  $U$  energy and make specific description of the impurity atom by applying additional orbital  $\varphi_d$  with its occupation number  $n_d$  and birth operator  $c_{d\sigma}^+$  matched it. Model Hamiltonian is formulated as follows:

$$H = \sum_{k,\sigma} \varepsilon_k n_{k\sigma} + U n_{d\uparrow} n_{d\downarrow} + E_d (n_{d\uparrow} + n_{d\downarrow}) + \sum_{k,\sigma} V_{dk} c_{d\sigma}^+ c_{k\sigma},$$

where, apart from free electrons and magnetic term with constant  $U$ , we have added tunneling  $V_{dk}$  term  $d - k$  which describes the effect of electron tunneling through the centrifugal barrier to convert orbital  $\varphi_d$  to one of the Friedel resonances.

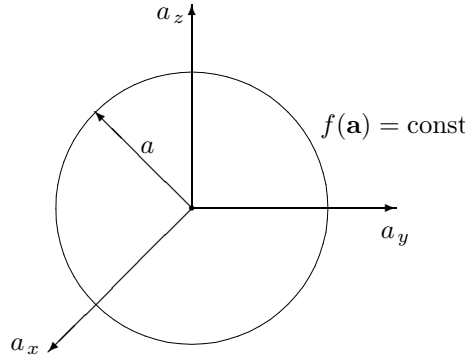
Before proceeding to other matters of this discussion, I would like to say some words about simplicity of the model which is actually more likely superficial than true. Any skills of modeling result in ability to give up actual and existent, but not significant aspects of the problem to be of risks for both the author and reader. As a matter of fact, the author can give up anything of importance; the reader to be in possession of excessively delicate experimental techniques or to be excessively punctual in calculations may oversimplify the conceptualized model which is chiefly targeted to demonstration of any specific potential.

As referred to the science style, one of my profound beliefs is that quantum mechanics and statistical physics are in principal so simple to make many facts, the Nature present with, look like improbable and just the only demonstration of the reasonable interpretable mechanism leaves no doubt in adequacy of the interpretation. Moreover, it concerns the cases when any unexpected phenomena are correctly predicted; particularly those related to low-density moment life-span correlation, previously described orbital-moment quenching, as respects to all the  $d$  – level impurities, or negative free-electron exchange polarization subject to further discussion. More often than not, such simplified model clears up the real nature of the phenomenon, and thereby any 'ab initio' value calculated for the purpose of specific cases, which, if they are actually correct, often contain a great number of elements that more likely conceal than disclose the truth. Thus, the possibility of providing precise calculation or measurement of something may be more likely an obstacle than an advantage, since anything under measurement or calculation occurs to be unessential from the standpoint of phenomenon mechanism investigation. After all, any ideal calculation just copies Nature, but not explains it".

How it comes that Hamiltonian is named the model one? Because it is nearly phenomenological - i.e. it is not precisely calculable. And if there are a few opportunities to make any calculations, then it is fine. This Hamiltonian may be applied for describing the experiment under study.

## 2. Anisotropy

Now, it is time to describe isotropic distribution [3]. We have prescribed function  $f = f(\mathbf{a})$ , i.e. value  $f$  depends on vector  $\mathbf{a}$ . If value  $f$  depends on modulus  $a$  of this vector only, the distribution concerned is called isotropic, i.e. it may be formulated as  $f = f(a)$ . This kind of isotropy may be represented graphically (see Fig. 1). We will plot a sphere of radius  $a$  centered in the origin of coordinates. So, value  $f$  will remain equal at any point of this sphere, providing that  $f = f(\mathbf{a})$  is the isotropic function. Any other  $f = f(\mathbf{a})$  function will be referred to the anisotropy one.



*Fig. 1. Isotropic function.*

Now, we will consider the example of the anisotropic function. We will plot two vectors. One of them will be an arbitrary vector  $\mathbf{a}$  and the other one will be rated as equal, but opposite in its direction  $-\mathbf{a}$ . Two such vectors are shown in Fig. 2. So, if it is appeared that function values fail matching in the points concerned, i.e.  $f(\mathbf{a}) \neq f(-\mathbf{a})$ , this function will be called the anisotropic one. Some exhaustive examples of the anisotropic function may be additionally described, but, as a matter of fact, the information provided is sufficient for understanding.

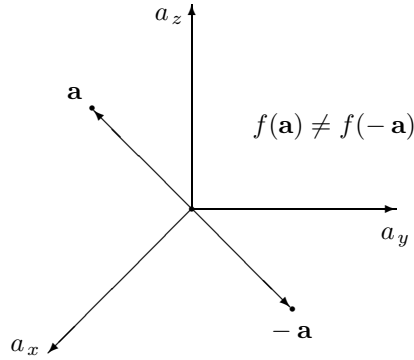


Fig. 2. Example of anisotropic function.

### 3. Fermi – Dirac function

Now, we will consider formulation of Fermi – Dirac function and review its meaning [4]. In the first place, we are chiefly speaking about electrons taking account to their large and very large number, for example solid-state body electrons. Secondly, electrons are subject to Pauli principle, according to which maximum one electron may be in a particular state. This function has one of the following simplest formulations:

$$\bar{n}(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)} + 1}, \quad (3.1)$$

where  $\bar{n}(\varepsilon)$  is an average number of  $\varepsilon$  – energy electrons,  $\beta = 1/k_B T$  – reciprocal temperature,  $\mu$  – chemical potential. For the function curve, see Fig. 3.

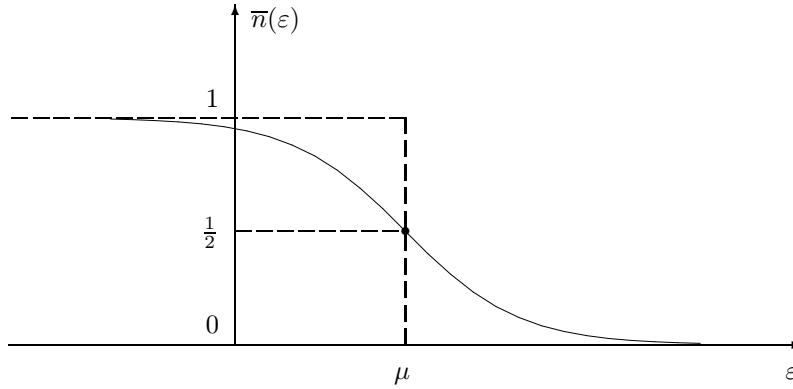


Fig. 3. Fermi – Dirac function.

Fermi – Dirac function shall be derived for the purpose of electron interacting system only. Since electrons interact to each other, just an approximate equation may be derived for function  $\bar{n}(\varepsilon)$ . The simplest one is mean field approximation in scope of which it is defined that double probability is equal to the product of single probabilities. This kind of approximation is known as statistical independence. As referred to the mean field approximation, the following equation is obtained:

$$\bar{n}(\varepsilon) = \frac{1}{e^{\beta(\bar{\varepsilon}-\mu)} + 1}, \quad (3.2)$$

where  $\bar{\varepsilon}$  is average electron energy which is equal to the sum of kinetic energy  $\varepsilon$  and energy of interaction of this electron with other electrons.

Now, let us remember that in specific representation a particular state of electron is described by wave function  $\psi_{\mathbf{k}n}(\mathbf{r})\chi_{\sigma}(\xi)$ , where  $\mathbf{k}$  is a wave vector,  $n$  and  $\sigma$  – other numbers, which in combination with the wave vector make it possible to define the state of electron. The wave vector is linked with an

electron momentum by the simple relation  $\mathbf{p} = \hbar \mathbf{k}$ . Now, without going into any details, we formulate the equation (3.2) as follows:

$$\ln \frac{1 - w_{\mathbf{k}}}{w_{\mathbf{k}}} = \beta (\bar{\varepsilon}_{\mathbf{k}} - \mu), \quad (3.3)$$

where  $w_{\mathbf{k}} = \bar{n}(\varepsilon_{\mathbf{k}})$  is probability of  $\mathbf{k}$  vector occupation probability;  $\bar{\varepsilon}_{\mathbf{k}}$  is single electron mean energy;  $\varepsilon_{\mathbf{k}}$  is electron kinetic energy. By this means as follows:

$$\bar{\varepsilon}_{\mathbf{k}} = \varepsilon_{\mathbf{k}} + \sum_{\mathbf{k}'} \varepsilon_{\mathbf{k}\mathbf{k}'} w_{\mathbf{k}'} . \quad (3.4)$$

The addend in this formula is the mean energy of interaction of an electron with other electrons. Summand  $\varepsilon_{\mathbf{k}\mathbf{k}'}$  is the energy of interaction of two electrons with wave vectors  $\mathbf{k}$  and  $\mathbf{k}'$ .

#### 4. Model Hamiltonian

Equation (3.3) contains its nonlinearly unknown function  $w_{\mathbf{k}}$ . Now, for the purpose of the above function, it is necessary to define the electron interaction energy  $\varepsilon_{\mathbf{k}\mathbf{k}'}$ . Electrons interact with each other under the Coulomb repulsive potential. But there is rather large number of ions and other electrons applied for interaction in the solid body. Need to say that it is very hard to calculate the rate of electron interaction energy. Therefore, we will use the model Hamiltonian. We will assume that [5]

$$\varepsilon_{\mathbf{k}\mathbf{k}'} = I \delta_{\mathbf{k}+\mathbf{k}'}, \quad (4.1)$$

where  $\delta_{\mathbf{k}+\mathbf{k}'}$  is Kronecker delta,  $I$  is  $\mathbf{k}$  and  $\mathbf{k}' = -\mathbf{k}$  wave vector and electron interaction energy. As provided in scope of our model, only those valence electrons are repulsive, which are able to surmount the crystal area at equal opposite direction velocities.

With the model formula applied (4.1) the average electron energy (3.4) is formulated by the equation as follows:

$$\bar{\varepsilon}_{\mathbf{k}} = \varepsilon_{\mathbf{k}} + I w_{-\mathbf{k}} . \quad (4.2)$$

According to this formula, the higher  $\mathbf{k}$  vector electron energy, the more probability  $w_{-\mathbf{k}}$  of  $-\mathbf{k}$  wave vector occupation. Hereby, the  $-\mathbf{k}$  wave vector electron somewhat affects the electron forcing it out of  $\mathbf{k}$  wave vector state.

We will insert formula (4.2) in equation (3.3). We will gain the following formula:

$$\ln \frac{1 - w_{\mathbf{k}}}{w_{\mathbf{k}}} = \beta (\varepsilon_{\mathbf{k}} + I w_{-\mathbf{k}} - \mu) . \quad (4.3)$$

Thus, the equation containing two function values  $w_{\mathbf{k}}$  and  $w_{-\mathbf{k}}$  is produced.

#### 5. Isotropic and anisotropic electron distribution

If you ask why it has to do with anisotropy, it may be confirmed by the equation (4.3) which exhibits solution of anisotropic function  $w_{\mathbf{k}}$  subject to condition of  $w_{-\mathbf{k}} \neq w_{\mathbf{k}}$ . In this equation we will substitute vector  $\mathbf{k}$  for vector  $-\mathbf{k}$ . If to consider that kinetic energy is the isotropic function, i.e.  $\varepsilon_{-\mathbf{k}} = \varepsilon_{\mathbf{k}}$ , we will formulate the following equation:

$$\ln \frac{1 - w_{-\mathbf{k}}}{w_{-\mathbf{k}}} = \beta (\varepsilon_{\mathbf{k}} + I w_{\mathbf{k}} - \mu) . \quad (5.1)$$

Equations (4.3) and (5.1) produce the system containing two unknown  $w_{\mathbf{k}}$  and  $w_{-\mathbf{k}}$ . At the same time, it is clear enough that probability  $w_{\mathbf{k}}$  is the composite  $\mathbf{k}$  vector function, where electron kinetic energy is applied as an intervening variable  $\varepsilon_{\mathbf{k}}$ :

$$w_{\mathbf{k}} = w(\varepsilon_{\mathbf{k}}) . \quad (5.2)$$

Combined equations (4.3) and (5.1) exhibit their anisotropic solution subject to the condition as follows:

$$w_{-\mathbf{k}} = w_{\mathbf{k}} . \quad (5.3)$$

Using this equation we will eliminate  $w_{-\mathbf{k}}$  from the combined equations (4.3) and (5.1). We will find solution of isotropic distribution function by applying the following equation:

$$\ln \frac{1 - w_{\mathbf{k}}}{w_{\mathbf{k}}} = \beta (\varepsilon_{\mathbf{k}} + I w_{\mathbf{k}} - \mu). \quad (5.4)$$

There are also some anisotropic distribution functions that fall out of formula (5.3) when specific wave vector values are applied:

$$w_{-\mathbf{k}} \neq w_{\mathbf{k}}.$$

Such kind of electron state distribution anisotropy may occur even when no external field is available. While applying forms:

$$w_{-\mathbf{k}} = w_1(\varepsilon_{\mathbf{k}}), \quad w_{\mathbf{k}} = w_2(\varepsilon_{\mathbf{k}}), \quad (5.5)$$

we may formulate equations (4.3) and (5.1) by the method as follows:

$$\left. \begin{aligned} \ln \frac{1 - w_1}{w_1} &= \frac{4}{\tau} (\epsilon + w_2), \\ \ln \frac{1 - w_2}{w_2} &= \frac{4}{\tau} (\epsilon + w_1), \end{aligned} \right\} \quad (5.6)$$

where

$$\epsilon = \frac{\varepsilon - \mu}{I}, \quad \tau = \frac{4\theta}{I}.$$

The following functions remain unknown in the combined equations (5.6):

$$w_1 = w_1(\varepsilon) \quad \text{и} \quad w_2 = w_2(\varepsilon).$$

If electrons have isotropic wave vector distribution, it is necessary to insert  $w_1 = w_2 = w_0$  in the combined equation (5.6). In this case, the equation gained may be formulated by the method as follows:

$$\epsilon = \frac{\tau}{4} \ln \frac{1 - w_0}{w_0} - w_0. \quad (5.7)$$

This equation states specific dependence of  $w_0 = w_0(\epsilon)$  with various temperature values graphically represented in Fig. 4 in the form of monotonically decreasing curves.

If electrons have anisotropic wave vector distribution, probabilities  $w_1$  and  $w_2$  in the combined equations (5.6) shall be considered as various functions  $w_1 = w_1(\epsilon)$  and  $w_2 = w_2(\epsilon)$  subject to energy  $\epsilon$ . To determine these dependences, we will introduce new variables  $d$  and  $s$  applying the relations as follows

$$w_2 - w_1 = d, \quad w_1 + w_2 = 1 + s. \quad (5.8)$$

Without loss of generality we will assume that nonnegative difference  $d$  of two distribution function values  $w_1$  and  $w_2$  is  $d \geq 0$ . At the same time,  $d$  remains equal to  $d \in [0, 1]$ . Value  $s$  may possess the values within the range of  $-1$  to  $1$ :  $s \in [-1, 1]$ . We will determine the equalities (5.8), as regards the probabilities  $w_1$  and  $w_2$ :

$$w_1 = \frac{1}{2} (1 + s - d), \quad w_2 = \frac{1}{2} (1 + s + d). \quad (5.9)$$

We will transform the combined equations (5.6) by applying the formulas (5.9). We will firstly subtract specific equation from one of the combined equations and then we will add the equations. As a result, we will obtain the following combined equations:

$$\frac{(1 + d)^2 - s^2}{(1 - d)^2 - s^2} = e^{4d/\tau}, \quad \epsilon = \frac{\tau}{8} \ln \frac{(1 - s)^2 - d^2}{(1 + s)^2 - d^2} - \frac{1}{2} (1 + s). \quad (5.10)$$

The first equation of the combined ones may be easily resolved against  $s$ :

$$s(d) = \pm \sqrt{\frac{(1 - d)^2 e^{4d/\tau} - (1 + d)^2}{e^{4d/\tau} - 1}}.$$

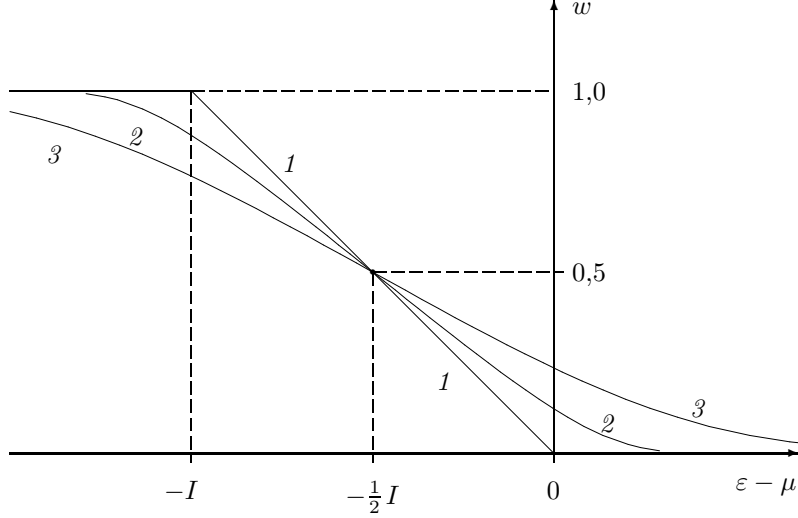


Fig. 4. Isotropic function of distribution of conductivity electron energy at various temperature values  $\tau$ : 1 -  $\tau = 0$ ; 2 -  $\tau = 0,25$ ; 3 -  $\tau = 0,8$ .

As concerns the relations (5.9), probabilities  $w_1$  and  $w_2$  may be also considered as  $d$  functions:  $w_1 = w_1(d)$ ,  $w_2 = w_2(d)$ . With the second equation of the combined ones (5.10) applied, we may express electron energy  $\epsilon$  in terms of parameter  $d$ . Using the dependences produced specific graphs of functions  $w_1 = w_1(\epsilon)$  and  $w_2 = w_2(\epsilon)$  may be easily plotted for various temperature values. For the plotted curve, see Fig. 5.

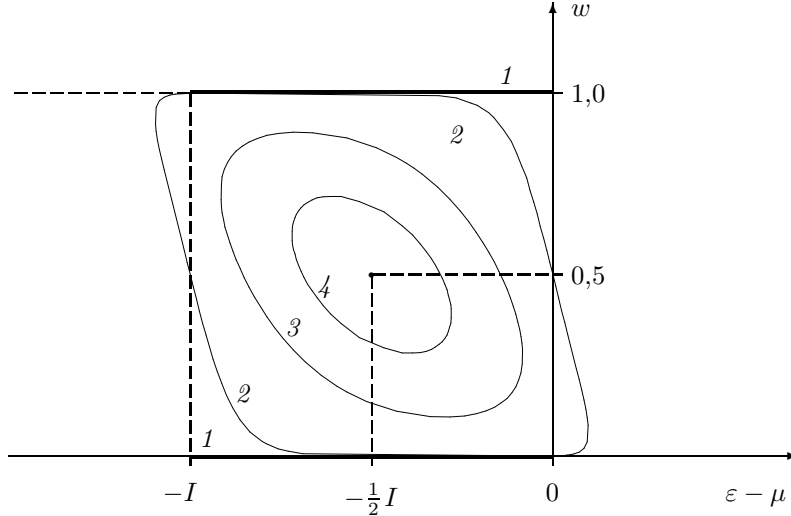


Fig. 5. Anisotropic function of distribution of conductivity electron energy at various temperature values  $\tau$ : 1 -  $\tau = 0$ ; 2 -  $\tau = 0,25$ ; 3 -  $\tau = 0,8$ ; 4 -  $\tau = 0,95$ .

The pattern of distribution of electrons by their states depends on their relation between metal temperature  $T$  and critical temperature:

$$T_c = \frac{I}{4 k_B}.$$

At the temperature of  $T \geq T_c$  the distribution function  $w = w(\epsilon)$  is single-valued and satisfies the condition (5.3), as respects all the  $\epsilon$  energy values. At the temperature of  $T < T_c$  the energy is limited by  $(\epsilon_1, \epsilon_2)$  with function  $w = w(\epsilon)$  possessing any of three values at every point of the limit, particularly  $w_1(\epsilon) < w_0(\epsilon) < w_2(\epsilon)$ . Being out of the aforesaid limit, the distribution function  $w = w(\epsilon)$  possesses only a single value  $w_0(\epsilon)$ . Thus, equation (5.4) is resolved by applying function  $w_{\mathbf{k}} = w_0(\epsilon_{\mathbf{k}})$  to describe isotropic wave vector electron distribution.

## 6. Order parameter

At  $T < T_c$  some kind of anisotropic wave vector electron distribution may occur in the narrow layer  $S$  under Fermi surface  $\varepsilon_{\mathbf{k}} = \mu$ . This kind of distribution is formulated by

$$w_{\mathbf{k}} = w_2(\varepsilon_{\mathbf{k}}), \quad w_{-\mathbf{k}} = w_1(\varepsilon_{\mathbf{k}}) \quad (6.1)$$

subject to  $\varepsilon \in (\varepsilon_1, \varepsilon_2)$ . Difference  $d = w_2 - w_1$  of two anisotropic electron distribution function values possesses the largest value  $d_{max}$  subject to  $\varepsilon = 0, 5$ . In this case,  $w_0 = 0, 5$  and  $s = 0$ . We will determine difference  $d_{max}$  from temperature  $\tau$  by applying  $s = 0$  in the first equation (5.10):

$$\frac{2 d_{max}}{\tau} = \ln \frac{1 + d_{max}}{1 - d_{max}}. \quad (6.2)$$

For dependence curve, see Fig. 6.

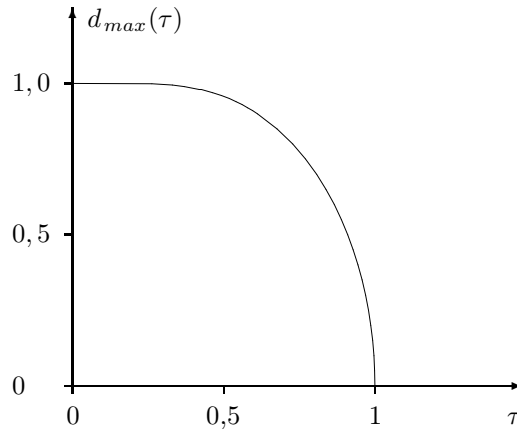


Fig. 6. Electron distribution anisotropy parameter  $d_{max}$ , as  $\tau$  temperature function.

## 7. Electron $T = 0$ distribution

At  $T = 0$  the isotropic distribution function is formulated as follows:

$$w_{\mathbf{k}} = \begin{cases} 1 & \text{at } \varepsilon_{\mathbf{k}} \leq \mu - I, \\ -\frac{1}{I}(\varepsilon_{\mathbf{k}} - \mu) & \text{at } \mu - I < \varepsilon_{\mathbf{k}} < \mu, \\ 0 & \text{at } \varepsilon_{\mathbf{k}} \geq \mu. \end{cases} \quad (7.1)$$

As for the anisotropic distribution, it is formulated as follows:

$$w_{\mathbf{k}} = 1 \quad \text{at} \quad \varepsilon_{\mathbf{k}} \leq \mu - I,$$

$$w_{\mathbf{k}} = 1, \quad w_{-\mathbf{k}} = 0 \quad \text{or} \quad w_{\mathbf{k}} = 0, \quad w_{-\mathbf{k}} = 1 \quad (7.2)$$

$$\text{at } \mu - I < \varepsilon_{\mathbf{k}} < \mu, \quad (7.3)$$

$$w_{\mathbf{k}} = 0 \quad \text{at} \quad \varepsilon_{\mathbf{k}} \geq \mu.$$

As provided by the formula (7.2), layer  $S$  may be determined under Fermi surface by the inequality (7.3), in which the electrons have anisotropic wave vector distribution, i.e. one of the both  $\mathbf{k}$  and  $-\mathbf{k}$  wave vector states in this layer is free and another one is occupied. For the function curves, see Fig. 7. Apparently, electron distribution function obtains its three  $S$  layer values. And what is the matter it stands for? The answer is in the value of energy, the isotropic or anisotropic distribution electrons exhibit. The electrons gain their steady state when they have the lowest energy.

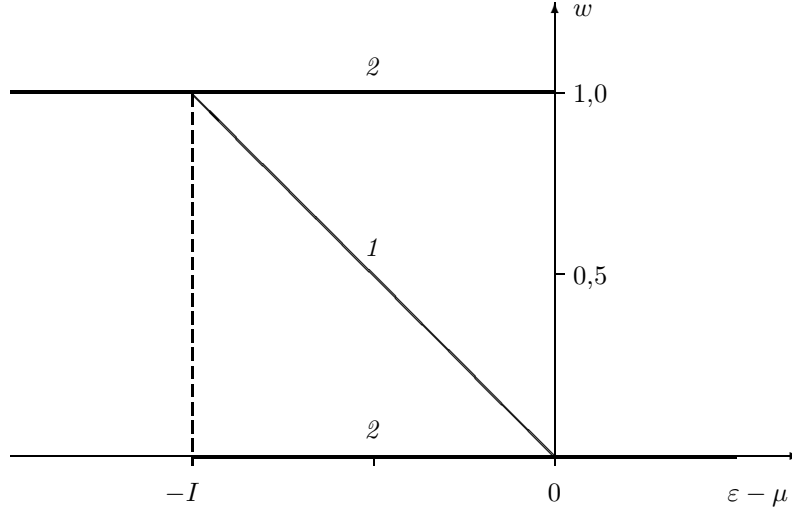


Fig. 7. Isotropic and anisotropic distribution of conductivity electrons depending on their kinetic energy at temperature  $\tau = 0$ : 1 - isotropic distribution, 2 - anisotropic distribution

## 8. Superconductivity. Energy of states

In scope of normalization conditions, the average itinerant electron velocity may be defined by the formula as follows:

$$\mathbf{v} = \frac{G \hbar}{m \overline{N}} \sum_{\mathbf{k}} \mathbf{k} w_{\mathbf{k}}, \quad (8.1)$$

where  $G = 2n_s$  is a number of states in one node and  $\overline{N}$  - mean number of conductivity electrons in a crystal. If the distribution function is isotropic, mean electron velocity  $\mathbf{v}$  gets equal to zero. Formula (8.1) may assign specific nonzero electron ordered motion velocity values to some anisotropic distribution functions, i.e. these distribution functions are applicable for defining electric current. If there are steady-state currents to exist with no external fields available, than such itinerant electron system states shall be considered as the superconductive ones [6-8].

We will assume that the state of electron gas is described by the anisotropic distribution function (6.1) or (7.2). In this case, mean electron ordered motion velocity modulus  $\mathbf{v}$  may assign any value rated from zero to certain  $v_{max}$ . The mean velocity will be equal to zero, providing that free pairs and those occupied by wave vectors  $\mathbf{k}$  and  $\mathbf{k}$  are chaotically distributed within layer  $S$ . If all the states concerned are occupied in one half of the layer (this is to say at  $k_x > 0$ ) and free in another half of the layer (at  $k_x < 0$ ), the electrons will gain their maximum ordered motion velocity. The value assigned by the mean electron velocity is defined by the nature of initial electron gas state. If the pattern of anisotropic wave vector electron distribution is rather steady with respect to small environment variations, the electron velocity value will survive for ages. This means that the metal concerned was able to gain its specific superconductive characteristics.

Now, we will calculate the energy the isotropic and anisotropic distribution electrons exhibit. We will apply the normalization condition formulated as follows:

$$G \sum_{\mathbf{k}} w_{\mathbf{k}} = \overline{N}. \quad (8.2)$$

Mean field approximation electron energy takes on the following form:

$$\overline{E} = G \sum_{\mathbf{k}} \left( \varepsilon_{\mathbf{k}} w_{\mathbf{k}} + \frac{1}{2} I w_{\mathbf{k}} w_{-\mathbf{k}} \right). \quad (8.3)$$



We will approximate dependence of electron kinetic energy  $\varepsilon_{\mathbf{k}}$  from wave vector  $\mathbf{k}$  by applying the formula as follows:

$$\varepsilon_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m}, \quad (8.4)$$

where  $m$  is effective itinerant electron mass. As provided by this formula, any electron kinetic energy shall be counted from the band bottom to be also called bottom of conduction band, i.e.  $\varepsilon_{\mathbf{k}=0} = 0$ .

To simplify calculations, instead of  $\mathbf{k}$  summing we will produce integration by  $\varepsilon$  electron kinetic energy. By applying the dependence (8.4) we will obtain the following symbolic equation:

$$G \sum_{\mathbf{k}} \dots = A \overline{N} \int_0^\infty \dots \sqrt{\varepsilon} d\varepsilon,$$

where

$$A = \frac{G m \sqrt{2m} V}{2 \pi^2 \hbar^3 \overline{N}}, \quad \varepsilon = \frac{\hbar^2 k^2}{2m}, \quad d\varepsilon = \frac{\hbar^2 k dk}{m}.$$

The upper integration limit may be equal to  $\infty$ , since the occupational probability of states which energy  $\varepsilon$  is specified at the ceiling of conduction band is actually equal to zero. Now, we will formulate the normalization condition (8.2) by the method as follows:

$$A \int_0^\infty w(\varepsilon) \sqrt{\varepsilon} d\varepsilon = 1. \quad (8.5)$$

As for the isotropic distribution electron energy, we will formulate the following formula:

$$\overline{E}^{(i)} = A \overline{N} \int_0^\infty \left( \varepsilon + \frac{1}{2} I w(\varepsilon) \right) w(\varepsilon) \sqrt{\varepsilon} d\varepsilon. \quad (8.6)$$

If the isotropic electron distribution function applied at  $T = 0$  is formulated according to (7.1), the equations (8.5) and (8.6) take on the following form:

$$A \int_0^{\mu-I} \sqrt{\varepsilon} d\varepsilon + A \int_{\mu-I}^\mu \frac{\mu - \varepsilon}{I} \sqrt{\varepsilon} d\varepsilon = 1,$$

$$\overline{E}_o^{(i)} = A \overline{N} \int_0^{\mu-I} \left( \varepsilon + \frac{1}{2} I \right) \sqrt{\varepsilon} d\varepsilon + \frac{A \overline{N}}{2I} \int_{\mu-I}^\mu (\mu^2 - \varepsilon^2) \sqrt{\varepsilon} d\varepsilon.$$

Since we apply the small parameter

$$\lambda = \frac{I}{\varepsilon_F},$$

where

$$\varepsilon_F = \left( \frac{3}{2A} \right)^{2/3}$$

– refers to Fermi energy, we will define that the chemical potential and isotropic wave vector distribution electron energy at  $T = 0$  take on the forms as follows:

$$\mu_o = \varepsilon_F \left( 1 + \frac{1}{2} \lambda - \frac{1}{48} \lambda^2 + \dots \right),$$

$$\overline{E}_o^{(i)} = \overline{N} \varepsilon_F \left( \frac{3}{5} + \frac{1}{2} \lambda - \frac{1}{16} \lambda^2 + \dots \right). \quad (8.7)$$

We will assume that the anisotropic wave vector electron distribution at  $T = 0$  is defined by the function as follows:

$$w_{\mathbf{k}} = \begin{cases} 1 & \text{at } \varepsilon_{\mathbf{k}} \leq \mu - I, \\ 1 & \text{at } \mu - I < \varepsilon_{\mathbf{k}} < \mu, \ k_x > 0 \\ 0 & \text{at } \mu - I < \varepsilon_{\mathbf{k}} < \mu, \ k_x < 0 \\ 0 & \text{at } \varepsilon_{\mathbf{k}} \geq \mu. \end{cases} \quad (8.8)$$

As provided by the above formula, only one half of the  $k_x > 0$  states may be referred to as the occupied ones to occur in layer  $S$  above the Fermi surface, which thickness  $\delta k$  is proportional to interaction parameter  $I$ . For anisotropic distribution pattern, see Fig. 8.

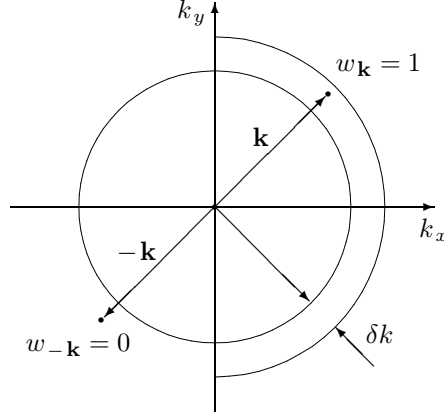


Fig. 8. Anisotropic distribution function at  $T = 0$ .

Here, the normalization condition gives rise to the following equation:

$$A \int_0^{\mu-I} \sqrt{\varepsilon} d\varepsilon + \frac{1}{2} A \int_{\mu-I}^{\mu} \sqrt{\varepsilon} d\varepsilon = 1.$$

As for electron energy, it may be calculated by the formula as follows:

$$\overline{E}_o^{(s)} = A \overline{N} \int_0^{\mu-I} \left( \varepsilon + \frac{1}{2} I \right) \sqrt{\varepsilon} d\varepsilon + \frac{1}{2} A \overline{N} \int_{\mu-I}^{\mu} \varepsilon \sqrt{\varepsilon} d\varepsilon.$$

As provided by the above calculation, the following formulation is obtained:

$$\begin{aligned} \mu &= \varepsilon_F \left( 1 + \frac{1}{2} \lambda - \frac{1}{16} \lambda^2 + \dots \right), \\ \overline{E}_o^{(s)} &= \overline{N} \varepsilon_F \left( \frac{3}{5} + \frac{1}{2} \lambda - \frac{3}{16} \lambda^2 + \dots \right). \end{aligned} \quad (8.9)$$

If occupied and free state pairs that match specific wave vectors  $\mathbf{k}$  and  $-\mathbf{k}$  will be distributed within layer  $S$  by any other way, the chemical potential and electron energy rating will remain the same.

The difference of electron energy values (8.7) and (8.9) will be formulated by the equation as follows:

$$\overline{E}_o^{(i)} - \overline{E}_o^{(s)} = \frac{\overline{N} I^2}{8 \varepsilon_F} > 0.$$

Thus, we get to the conclusion that the state of itinerant electrons described by the anisotropic distribution function is the primary one - i.e. the electron system specified in this condition is of the lowest energy.

Considering for the aforesaid about the anisotropic electron energy distribution we will plot the pattern of superconductive state, as shown in Fig. 9.

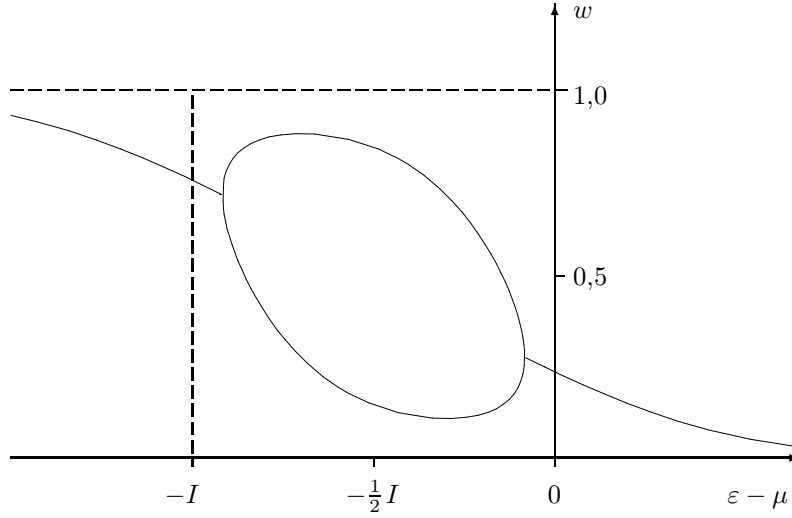


Fig. 9. Anisotropic distribution of energy  $\varepsilon$  conductivity electrons subject to the lowest energy  $\bar{E}$  at the temperature of  $\tau = 0.8$ .

### 9. Maximum superconductivity electron velocity at $T = 0$

Now, we will find ordered electron motion velocity in the state described by the distribution function (7.2) at the temperature of  $T = 0$ . For this purpose, we will substitute the wave vector sum specified in the formula (8.1) for the following integral:

$$\mathbf{v} = \frac{G \hbar V}{8 \pi^3 m \bar{N}} \int \mathbf{k} w_{\mathbf{k}} d^3 k. \quad (9.1)$$

If  $T = 0$ , the inner and outer radii of layer  $S$  shall be respectively equal to as follows:

$$k_1 = \frac{1}{\hbar} \sqrt{2 m (\mu - I)}, \quad k_2 = \frac{1}{\hbar} \sqrt{2 m \mu}.$$

In this case, the layer thickness is defined by the following formula:

$$\delta k = k_2 - k_1 = \frac{I}{\hbar} \sqrt{\frac{m}{2 \varepsilon_F}}. \quad (9.2)$$

If the states are filled with electrons in one half of layer  $S$ , but another one remains free, the rate of ordered electron motion velocity exhibits its maximum value

$$v_{max} = \frac{3 I}{4 \sqrt{2 m \varepsilon_F}}. \quad (9.3)$$

### 10. Electric current-forced superconducting state destroy effect

Superconducting state of the itinerant electrons is destroyed in the events when the current running over metal items exceeds its particular critical value (Silsbee effect). We will assume that specific homogeneous electric field with tension  $\mathbf{E}$  is produced inside the metal. Under the effect of this field the itinerant electrons will execute their ordered motion at the average velocity  $\mathbf{u}$  which direction agrees with another one that affecting force electron  $-e \mathbf{E}$ . In this case, electron state distribution function  $w_{\mathbf{k}}$  may be defined by the equation (3.3), where electron energy  $\bar{\varepsilon}_{\mathbf{k}}$  depends on wave vector  $\mathbf{k}$  in the following manner:

$$\bar{\varepsilon}_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \hbar \mathbf{k} \mathbf{u} + I w_{-\mathbf{k}}. \quad (10.1)$$

It is no easy matter to make exact solution of the equation (3.3). Therefore, we will consider its approximate solution only. We will assume that distribution function  $w_{\mathbf{k}}$  has the form as follows:

$$w_{\mathbf{k}} = f(\mathbf{k} - \mathbf{k}_o), \quad (10.2)$$

where  $w_{\mathbf{k}} = f(\mathbf{k})$  is the solution of the equation (3.3), providing that  $\mathbf{u} = 0$ ;

$$\mathbf{k}_o = \frac{m \mathbf{u}}{\hbar}.$$

Function  $w_{\mathbf{k}} = f(\mathbf{k})$  is equal to a unity almost at all the points occurred inside Fermi sphere:  $k < k_F$ , excluding the points at surface  $S$  of that sphere. If occurred outside Fermi sphere, function  $w_{\mathbf{k}} = f(\mathbf{k})$  almost everywhere is equal to zero. Field  $D$  containing nonzero function (10.2) is limited by the sphere with radius  $k_F$ , which center  $C$  is displaced off the origin of coordinates  $O$  by vector  $\mathbf{k}_o$ . In other words, the wave vectors that agree with occupied electronic states belong to field  $D$ . The theory under discussion is applied to superconductivity to be due to interaction of electrons, which wave vectors  $\mathbf{k}$  and  $-\mathbf{k}$  belong to spherical layer  $S$  with its radius equal to  $k_F$  and thickness - to  $\delta k$ . No anisotropy is created with the electrons distributed over their wave vectors, when the rate of displacement  $k_o$  of field  $D$  is so large that one half of layer  $S$  finds itself beyond this field. For arrangement of field  $D$  and layer  $S$ , see Fig. 10.

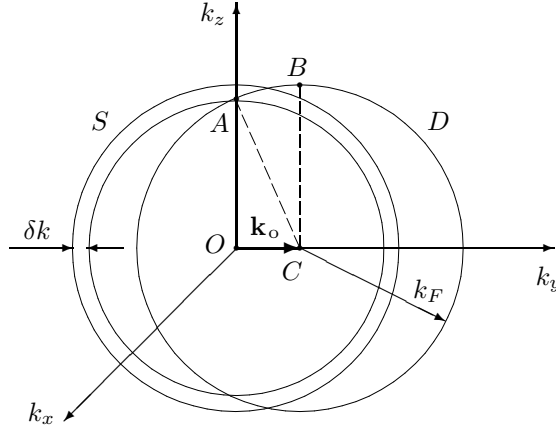


Fig. 10. Displacement of Fermi sphere under influence of electric field.

Point  $A$  belongs simultaneously to the displaced Fermi sphere and to the inner surface of layer  $S$ . Therefore,

$$AC = k_F, \quad AO = k_F - \delta k.$$

We will apply the rule of Pythagoras for  $AOC$  triangle. Now, we will gain the following formula:

$$k_F^2 = k_o^2 + (k_F - \delta k)^2.$$

It appears from this equation that superconductivity-force displacement  $k_o$  will be formulated as follows:

$$k_o = \sqrt{2 k_F \delta k}.$$

This formula may be transformed to:

$$k_o = \frac{\sqrt{2 m \delta \varepsilon}}{\hbar},$$

where  $\delta \varepsilon$  is energy width of the layer  $S$ . In this connection, the average current velocity will be calculated by the following equation:

$$u = \sqrt{\frac{2 \delta \varepsilon}{m}}. \quad (10.3)$$

By this means that the superconductive state of conduction electrons will be destroyed when the external electric field makes them moving to the same direction and produces the current, which density exceeds the following value:

$$j_{\kappa p} = e n u.$$

We will specify the relation of maximum superconductive current velocity  $v_{max}$  to the least current speed  $u$  destroying the superconductive state:

$$\frac{v_{max}}{u} = \frac{3}{8} \sqrt{\frac{\delta\varepsilon}{\varepsilon_F}}. \quad (10.4)$$

It is clear enough that the above relation is much less than unity  $v_{max} \ll u$ .

On cutting off the electric field and after thermodynamic equilibration the electrons shall transform to their superconductive state distributing as provided by the formula (7.2). Thereafter, the average electron velocity shall drop down to value  $v_{max}$  and superconductive current of maximum density  $j_{max}$  will run over the metal.

### 11. Mean energy dependence of kinetic energy

The rate of mean electron  $\bar{\varepsilon}_{\mathbf{k}}$  energy dependence of its kinetic energy  $\varepsilon_{\mathbf{k}}$  is defined by formula (4.2). As may be inferred from the above formula, the electron energy with wave vector  $\mathbf{k}$  depends on whether the  $-\mathbf{k}$  wave vector state is free or occupied. Electron energy  $\bar{\varepsilon}$  may be specified by the  $\varepsilon$  kinetic energy functional form as follows:

$$\bar{\varepsilon}(\varepsilon) = \varepsilon + I w_1(\varepsilon).$$

or the pattern of this function at various temperatures, see Fig. 11.

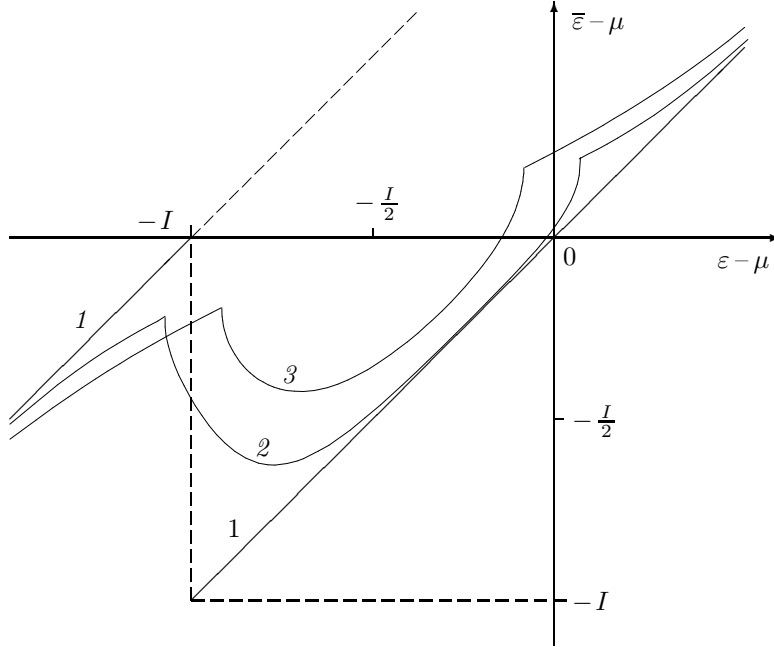


Fig. 11. Mean electron  $\bar{\varepsilon}$  energy dependence of its  $\varepsilon$  kinetic energy at various temperature values  $\tau$ : 1 -  $\tau = 0$ , 2 -  $\tau = 0,5$ , 3 -  $\tau = 0,8$ .

### 12. Conclusion

Thus, in scope of the theory discussed the microscopic superconductivity is due to anisotropic wave vector electron distribution. Normally, anisotropy is caused by electron repulsion effect in the  $\mathbf{k}$  and  $-\mathbf{k}$  wave vectors states. If applied to the matter of more complex nature, model Hamiltonian is formulated as follows:

$$\varepsilon_{\mathbf{k}\mathbf{k}'} = I \delta_{\mathbf{k}+\mathbf{k}'} - J \delta_{\mathbf{k}-\mathbf{k}'},$$

where  $J$  is attractive energy of two electrons in the conditions equal to wave vectors  $\mathbf{k} = \mathbf{k}'$ . This Hamiltonian is calculated in [9].

## Refereces

- [1] P.W.Anderson, Local Momentns and Localized States. Nobel Lecture, 8 December 1977.
- [2] P.W.Anderson, UFN, 1979, v.127, №.1, p.19.
- [3] Yu.I.Sirotn, M.P.Shaskolskaya, Basic Crystallophysics, M.: Nauka, 1979.
- [4] B.V.Bondarev, N.P.Kalashnikov, G.G.Spirin, General Physics Course, v.3, M.: Higher School, 2003.
- [5] B.V.Bondarev, Vestnik MAI, 1996, v.3, №.2, p.56.
- [6] H.Kamerlingh Onnes, Comm. Phys. Leb. Univ. Leiden, 1911, №122, p.13.
- [7] J.Bardeen, L.N.Cooper, J.R.Schrieffer, Phys.Rev., 1957, v.106, №1, p.162; 1957, v.108, №5, p.1175.
- [8] J.Schiffer, Superconductivity Theory, M.: Nauka, 1970.
- [9] B.V.Bondarev, Density Matrix Method in Quantum Theory of Cooperative Process, M.: Sputnik, 2001.